A New Joint Time Scale Method of Hydrogen Maser and Cesium Atomic Clock

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Abstract*—The fusion method of hydrogen maser and cesium atomic clock is mainly to study the effective combination of two different types of atomic clock to produce a more stable and accurate time scale. In this paper, the time scale fusion method of cesium clock and hydrogen maser based on Vondrak-Cepek filtering is studied, and the preliminary results are obtained. The clock difference data of three cesium clocks and three hydrogen masers are taken as research objects. First, the improved weighted average algorithm is used to generate the time scale of cesium clocks. the exponential filtering method is used to establish the time scale of hydrogen masers. Secondly, the key parameters of Vondark-Cepek combined filtering are selected according to the least square principle. Finally, the performance of the time scale of cesium clock ensemble is enhanced by the differential information of the time scale of hydrogen maser ensemble, and the time scale of hydrogen-cesium fusion is obtained. The results show that this method can effectively utilize the short-term stability of hydrogen maser and the long-term stability of the cesium clock. And the stability reaches 3.8e-15 in 1 h and 3.4e-15 in 5 d, which are superior to the corresponding time scales of the cesium clocks and the hydrogen masers, and the combined time scales of hydrogen masers and cesium clocks produced by AT1 and ALGOS algorithm respectively.

Keywords—Time Scale; Hydrogen maser; cesium atomic clock; Vondrak-Cepek combined filtering; stability

I. INTRODUCTION

Hydrogen maser and cesium atomic clock are two different types of atomic frequency standards, each with its own characteristics. Hydrogen masers have good short-term stability, while cesium atomic clocks have better long-term stability. They are the two commonly used timekeeping atomic clocks in laboratories all over the world. A time laboratory often seeks a time scale with good long-term and short-term stability [1]. At present, many studies are based on the combined timekeeping of hydrogen and cesium clock to make full use of the advantages of different types of atomic clocks [2]. In paper [3], take the hydrogen maser as a reference for the measurement of cesium atomic clock noise, the cesium atomic clock was filtered and the time scale of the cesium atomic clock ensemble was established. Then, the frequency and frequency drift of the hydrogen masers are estimated by using the time scale of the cesium clock ensemble as a reference, and the hydrogen maser deducted from the frequency drift parameter is used to calculate the time scale. In paper [4], reference time scale generated by cesium clocks is used to correct the rate and frequency drift of hydrogen masers, and short-term frequency jump of cesium clocks is corrected by time scale generated by hydrogen masers. In reference [5], the cesium atomic clock is deducted from the rate, and the hydrogen maser is deducted from the rate and frequency drift. The weighted average method is used to calculate the reference time scale to steer the hydrogen maser. In reference [6], reference time scale of cesium atomic clock is used to estimate the rate and frequency drift of hydrogen maser. Wavelet scale decomposition is used to reduce the influence of clock difference noise of maser, and finally the time scale of hydrogen maser ensemble is established. It is found that the phase white noise is the main noise when the hydrogen maser is used as a reference to measure the cesium atomic clock. After filtering by mathematical method, the short-term stability of the time scale is still affected by the noise of the cesium atomic clock. When the hydrogen maser breaks down, the reliability of the time scale cannot be guaranteed. Wavelet transform is used to reduce the noise of atomic clock. There are many parameters such as decomposition level and wavelet base selections need to be considered. These subjective factors directly determine the performance of wavelet filtering. In view of this, it is an algorithm that based on Vondrak-Cepek(V-C) combined smoothing method to generate the time scales of hydrogen masers and cesium clocks in order to make full use of the characteristics of the two types of atomic clocks. The short-term and long-term stability of V-C time scale of the two types of atomic clocks is better than that of the single clock. The stability of the time scales of union of hydrogen maser and cesium clock produced by the weighted average algorithm AT1 and ALGOS are also calculated. And the stability of V-C time scale is the best here. V-C filtering method has been well used in astronomy and UTC (Coordinated Universal Time) computation (combination of TW and PPP) [7,8]. This method is developed on the basis of the initial smoothing method proposed by Vondrak, Czech astronomer, in 1969. It is characterized by the fact that it does not need to know the changing rules of the data, nor need to provide a system fitting function or system equation. The data interval can be equal or unequal, and the measurement noise can be well suppressed

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without affecting the real signal in the data [9]. The essence of initial smoothing is compromise between absolutely smoothing and absolutely fitting of observation data. V-C combined smoothing method further adds the first derivative of the observed data, that is, the compromise in three cases of absolutely smoothing, absolutely fidelity and absolutely fitting of the first derivative. In this paper, the V-C combined smoothing method is used to make full use of the characteristics of two different types of atomic clocks of hydrogen maser and cesium clock, in order to produce a time scale with good long-term and short-term stability, and to improve the accuracy and stability of the time scale further. The clock difference data of three cesium atomic clocks and three hydrogen masers are used, and the performance of the V-C algorithm is analyzed.

II. V-C COMBINE SMOOTHING METHOD

We assume that the observation data of input sequence 1 is expressed as y_j', the corresponding time is x_j and the weight is p_j; the first derivative of input data 2 is expressed as $\bar{y}_k'$, the corresponding time is $\bar{x}_k$ and the weight is $\bar{p}_k$; the output sequence is expressed as y_i, the corresponding time is x_i and the weight is p_i. Define three quantities:

A. Algorithm Principles

The smoothness of a curve, such as formula (1)

$$S = \frac{1}{x_N - x_1}\int_{x_1}^{x_N} \varphi'''^2(x)dx \tag{1}$$

In the formula, the expression of $\varphi(x)$ is unknown, so it is necessary to estimate the third derivative of $\varphi'''(x)$ based on smooth data. The smooth curve between two points [x_{i+1}; y_{i+1}] and [x_{i+2}; y_{i+2}] is defined as a third-order Lagrangian polynomial $L_i(x)$ obtained from four adjacent points i, i+1, i+2, i+3, such as formula (2):

$$L_i(x) = \sum_{k=0}^{3}\Big(\prod_{\substack{j=0\\(j\neq k)}}^{3} \frac{(x - x_{i+j})}{(x_{i+k} - x_{i+j})}\Big)\, y_{i+k} \tag{2}$$

Its third derivative is formula (3):

$$L_i'''(x) = \sum_{k=0}^{3}\Big(6\prod_{\substack{j=0\\(j\neq k)}}^{3} \frac{1}{(x_{i+k} - x_{i+j})}\Big)\, y_{i+k} \tag{3}$$

Given that the third derivative between each pair of data points is a constant, it can be expressed as formula (4):

$$\begin{aligned} S &= \frac{1}{x_N - x_1}\sum_{i=1}^{N-3}\int_{x_{i+1}}^{x_{i+2}} L_i'''^2(x)\,dx \\ &= \sum_{i=1}^{N-3}(a_i y_i + b_i y_{i+1} + c_i y_{i+2} + d_i y_{i+3})^2 \end{aligned} \tag{4}$$

Here

$$a_i = \frac{6\sqrt{(x_{i+2} - x_{i+1})/(x_N - x_1)}}{(x_i - x_{i+1})(x_i - x_{i+2})(x_i - x_{i+3})},$$

$$b_i = \frac{6\sqrt{(x_{i+2} - x_{i+1})/(x_N - x_1)}}{(x_{i+1} - x_i)(x_{i+1} - x_{i+2})(x_i - x_{i+3})},$$

$$c_i = \frac{6\sqrt{(x_{i+2} - x_{i+1})/(x_N - x_1)}}{(x_{i+2} - x_i)(x_{i+2} - x_{i+1})(x_{i+2} - x_{i+3})},$$

$$d_i = \frac{6\sqrt{(x_{i+2} - x_{i+1})/(x_N - x_1)}}{(x_{i+3} - x_i)(x_{i+3} - x_{i+1})(x_{i+3} - x_{i+2})}$$

This definition of smoothing denotes that the ideal smoothing function (S = 0) is a quadratic function of time (its first derivative is a linear function).

1) The fidelity of smooth curves to observed values, such as formula (5)

$$F = \frac{1}{n}\sum_{i=1}^{N} p_i(y_i' - y_i)^2 \tag{5}$$

2) The fidelity of smooth curves to observed first derivatives, such as formula (6)

$$\bar{F} = \frac{1}{n}\sum_{i=1}^{N} \bar{p}_i(\bar{y}_i' - \bar{y}_i)^2 \tag{6}$$

The smoothing value of the first derivative $\bar{y}_i$ can be expressed by the value of the smoothing function y_i.

The first derivative of the Lagrange Polynomials $L_i(x)$ used to define S is expressed as $L_i'(x)$, such as formula (7).

$$L_i'(x) = A_i(x)y_i + B_i(x)y_{i+1} + C_i(x)y_{i+2} + D_i(x)y_{i+3} \tag{7}$$

Among them,

$$A_i(x) = \frac{\sum_{\substack{l=0\\(l\neq 0)}}^{2}\sum_{\substack{m=l+1\\(m\neq 0)}}^{3}(x - x_{i+l})(x - x_{i+m})}{(x_i - x_{i+1})(x_i - x_{i+2})(x_i - x_{i+3})},$$

$$B_i(x) = \frac{\sum_{\substack{l=0\\(l\neq 1)}}^{2}\sum_{\substack{m=l+1\\(m\neq 0)}}^{3}(x - x_{i+l})(x - x_{i+m})}{(x_{i+1} - x_i)(x_{i+1} - x_{i+2})(x_i - x_{i+3})},$$

$$C_i(x) = \frac{\sum_{\substack{l=0\\(l\neq 2)}}^{2}\sum_{\substack{m=l+1\\(m\neq 2)}}^{3}(x - x_{i+l})(x - x_{i+m})}{(x_{i+2} - x_i)(x_{i+2} - x_{i+1})(x_{i+2} - x_{i+3})},$$

$$D_i(x) = \frac{\sum_{\substack{l=0\\(l\neq 3)}}^{2}\sum_{\substack{m=l+1\\(m\neq 3)}}^{3}(x - x_{i+l})(x - x_{i+m})}{(x_{i+3} - x_i)(x_{i+3} - x_{i+1})(x_{i+3} - x_{i+2})},$$

In order to express $\bar{y}_i$ of the first derivative of smoothing function y_i, there is a certain freedom to choose four points around x_i at each time. The restriction of the following formula ensures that $\bar{y}_i$ is on a smooth curve.

1） In order to calculate $\bar{y}_1$, we use the values of time x_1, x_2, x_3, x_4; that is to say, we can get the formula (8).

$$\begin{gathered}\bar{a}_1 = A_1(x_1),\ \bar{b}_1 = B_1(x_1),\ \bar{c}_1 = C_1(x_1),\ \bar{d}_1 = D_1(x_1) \\ \bar{y}_1 = \bar{a}_1 y_1 + \bar{b}_1 y_2 + \bar{c}_1 y_3 + \bar{d}_1 y_4\end{gathered} \tag{8}$$

2） For the calculation of $\bar{y}_i$ in the first half of the input data, i=2，3，… N/2, use time x_{i-1}，x_i，x_{i+1}，x_{i+2}; that is formula (9)

$$\begin{gathered}\bar{a}_i = A_{i-1}(x_i),\ \bar{b}_i = B_{i-1}(x_i),\ \bar{c}_i = C_{i-1}(x_i),\ \bar{d}_i = D_{i-1}(x_i) \\ \bar{y}_i = \bar{a}_i y_{i-1} + \bar{b}_i y_i + \bar{c}_i y_{i+1} + \bar{d}_i y_{i+2}\end{gathered} \tag{9}$$

3） Calculation of $\bar{y}$ for the second half of input data, i= N/2+1, N/2+2，… N-1, using time x_{i-2}，x_{i-1}，x_i，x_{i+1}; i.e. formula (10)

$$\begin{gathered}\bar{a}_i = A_{i-2}(x_i),\ \bar{b}_i = B_{i-2}(x_i),\ \bar{c}_i = C_{i-2}(x_i),\ \bar{d}_i = D_{i-2}(x_i) \\ \bar{y}_i = \bar{a}_i y_{i-2} + \bar{b}_i y_{i-1} + \bar{c}_i y_i + \bar{d}_i y_{i+1}\end{gathered} \tag{10}$$

4） For the calculation of $\bar{y}_N$, the time x_{N-3}，x_{N-2}，x_{N-1}，x_N are used; that is, formula (11)

$$\begin{gathered}\bar{a}_N = A_{N-3}(x_N),\ \bar{b}_N = B_{N-3}(x_N),\ \bar{c}_N = C_{N-3}(x_N),\ \bar{d}_N = D_{N-3}(x_N) \\ \bar{y}_N = \bar{a}_N y_{N-3} + \bar{b}_N y_{N-2} + \bar{c}_N y_{N-1} + \bar{d}_N y_N\end{gathered} \tag{11}$$

Then $\bar{F}$ can be expressed as Formula (12)

$$\begin{aligned}\bar{F} = \frac{1}{n}\Big[&\bar{p}_1\big(\bar{y}_1' - \bar{a}_1 y_1 - \bar{b}_1 y_2 - \bar{c}_1 y_3 - \bar{d}_1 y_4\big)^2 \\ &+ \sum_{i=2}^{N/2}\bar{p}_i\big(\bar{y}_i' - \bar{a}_i y_{i-1} - \bar{b}_i y_i - \bar{c}_i y_{i+1} - \bar{d}_i y_{i+2}\big)^2 \\ &+ \sum_{i=N/2+1}^{N/2}\bar{p}_i\big(\bar{y}_i' - \bar{a}_i y_{i-2} - \bar{b}_i y_{i-1} - \bar{c}_i y_i - \bar{d}_i y_{i+1}\big)^2 \\ &+ \bar{p}_N\big(\bar{y}_N' - \bar{a}_N y_{N-3} - \bar{b}_N y_{N-2} - \bar{c}_N y_{N-1} - \bar{d}_N y_N\big)^2\Big]\end{aligned} \tag{12}$$

What we are looking for is the smoothing value y_i as an equilibrium under three different conditions, each setting will lead to different results:

- Curves need to be smoothed (minimizing S)
- Smoothing values need to be close to the observed values of the function (minimizing F)
- The first derivative of a smooth curve needs to be close to the observed value of the first derivative (minimizing $\bar{F}$)

By minimizing the above constraints, the adjustment is expressed in the following formula (13):

$$Q = S + \varepsilon F + \bar{\varepsilon}\bar{F} = min$$

$$\Rightarrow \frac{\partial Q}{\partial y_i} = 0\,, i = 1,2,\ldots N \qquad (13)$$

Among them, $\varepsilon \geq 0$, $\bar{\varepsilon} \geq 0$. The equilibrium degree of the three conditions can be determined by choosing the values of these two parameters: smoothing coefficients ε and $\bar{\varepsilon}$. The larger the value of smoothing coefficient is, the greater the weight of fidelity relative to the observed function value or its first derivative is, and the closer the smoothing value is to the observed value. The partial derivative of S for y_i can be simply expressed as formula (14):

$$\frac{\partial S}{\partial y_i} = 2(a_i \triangle_i + b_{i-1} \triangle_{i-1} + c_{i-2} \triangle_{i-2} + d_{i-3} \triangle_{i-3}) \qquad (14)$$

Among them,

$$\triangle_i = a_i y_i + b_i y_{i+1} + c_i y_{i+2} + d_i y_{i+3}$$

$$a_i = b_i = c_i = d_i = 0\,, i \leq 0 \text{ or } i \geq N-2$$

The partial derivative of function F is formula (15):

$$\frac{\partial F}{\partial y_i} = \frac{2p_i(y_i - y_i')}{n} \qquad (15)$$

B. Filter Parameter Design

V-C filtering method seeks equilibrium under three different conditions, and each different smoothing factor setting will lead to different results. Here, $\varepsilon \geq 0$ is the smoothing factor of the observation function F, and $\bar{\varepsilon} \geq 0$ is the smoothing factor of the first derivative $\bar{F}$. When $\varepsilon = 0$, $\bar{\varepsilon} \neq 0$, there is no unique solution to the system equation formula (13) in the case of only the first derivative of the observation. At this time, additional restrictive conditions are needed to solve the problem.

Here ε and $\bar{\varepsilon}$ are determined by frequency response method, such as formula (16). The formula reflects the relationship between the smoothing factor ε and $\bar{\varepsilon}$, the period P and the frequency response T and T', $0 \leq T \leq 1$, $0 \leq T' \leq 1$. Table 1 shows the relationship between the parameters above.

$$\varepsilon = \frac{(2\pi f)^6 T}{1-T}\,, \qquad \bar{\varepsilon} = \frac{(2\pi f)^4 T'}{1-T'} \qquad (16)$$

As can be seen from Table 1, when P is unchanged, ε increases with T, while when T is unchanged, ε decreases with P (the same as $\bar{\varepsilon}$). If we know that the frequency or period of the signal which need to be suppressed, we can fix the parameter and adjust the values of T, T' and ε and $\bar{\varepsilon}$ to find the best combination. Our goal is to fuse the generation time scale of cesium clocks with the time scale of hydrogen masers. The short-term fluctuation of cesium clocks is large, and the main performance of cesium clocks is white noise within 1d. So we set f as the frequency of white noise e-12. The purpose is to effectively suppress the white noise of cesium clock ensemble. Therefore, for the cesium clock ensemble is T=0.3, and for the hydrogen maser ensemble T'=0.99. In addition, the method can weigh the input data at each time according to the uncertainty. Since the data used in this paper are assumed to have the same uncertainties, all data are given equal weights in the next calculation.

TABLE I. RELATIONS BETWEEN FREQUENCY RESPONSES (T AND T') AND SMOOTHING FACTOR (ε AND $\bar{\varepsilon}$) AND PERIOD P

		T/T'		
		0.30	0.50	0.99
P=0.5	ε	1690000	3940000	390000000
	$\bar{\varepsilon}$	10700	24900	2470000
P=1	ε	26400	61500	6090000
	$\bar{\varepsilon}$	668	1560	154000

III. RESULT ANALYSIS

The original data used in this paper are all from three cesium atomic clocks and three hydrogen masers randomly selected from the timekeeping Laboratory of the National Time Service Center , Chinese Academy of Sciences. The clock difference data from January 1, 2021 (MJD = 59215) to February 1, 2021 (MJD = 59246) were selected to calculate. Firstly, weighted average algorithm AT1 and ALGOS are used to calculate the combined time scale of hydrogen maser and cesium clocks, respectively. Secondly, the time scale is established based on V-C combined smoothing(V-C) method, and the frequency response and smoothing parameters are selected according to the above methods. T=0.3， T'=0.99， ε=1690000， $\bar{\varepsilon}$=154000. Three Cesium atomic clocks and Hydrogen masers are calculated by improved weighted average algorithm and exponential filtering time scale algorithm to generate the time scale Cs WE and Hm EF. Three Cesium atomic clocks are calculated by exponential filtering time scale algorithm to generate the time scale Cs EF. At the same time, AT1 and ALGOS algorithm are used to calculate the joint time scale of six clocks. And the results are shown in Fig. 1. Fig.2 is the residual distributions of the combined smoothing results and the original data, respectively. The Allan deviations of V-C, AT1, ALGOS, Cs WE, Hm EF and Cs EF are shown in Table 2.

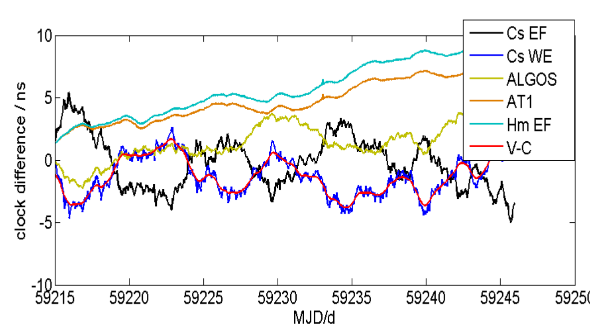

Fig. 1. Comparison of time scale results

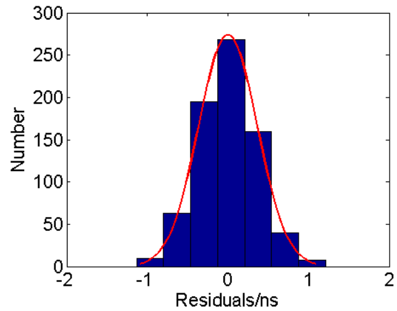

Fig. 2. Residual distribution of V-C fusion results and Cs WE

V-C combined smoothing method is used to fuse the time scale of cesium clocks and hydrogen masers. Compared the curve in Fig. 1, the results of V-C fusion are obviously smoother than those of AT1 and ALGOS combination. From Fig. 1, we can see that the time scale of V-C fusion results is very close to Cs WE, and RMSE = 0.129. This shows that the V-C fusion results curve can reflect the data details of Cs WE and has high fidelity. From Fig. 2, we can see that the time scale residuals of the cesium clock ensemble and V-C combined smoothing result are fluctuated up and down at 0, and the residuals obey the normal distribution, which conforms to the nature of white noise. Table 2 show that the Allan deviation of time scale of V-C result is 3.89e-15 in 4h and 3.4e-15 in 5d, which are smaller than the corresponding indexes of AT1, ALGOS, Cs WE, Cs EF and Hm EF. That is to say, the stability of V-C fusion result is better than the time scale stability of the two types of atomic clocks, and it is better than the time scale stability of AT1 and ALGOS. Therefore, V-C combined smoothing method can effectively utilize the excellent short-term stability of hydrogen masers and the excellent long-term stability characteristics of cesium clocks, and produce good time scales with both long-term and short-term stability.

TABLE II. STABILITY OF V-C, AT1, ALGOS, CS WE, CS EF AND HM EF

Averaging time/ns	ADEV					
	V-C	AT1	ALGOS	Cs WE	Cs EF	Hm EF
3.60e+12	3.84e-15	6.25e-15	3.37e-14	7.86e-14	7.88e-14	6.33e-15
7.20e+12	3.14e-15	3.34e-15	2.39e-14	5.27e-14	5.53e-14	3.33e-15
1.44e+13	3.16e-15	3.04e-15	1.68e-14	4.10e-14	4.25e-14	3.17e-15
2.88e+13	3.21e-15	3.68e-15	1.12e-14	2.51e-14	2.70e-14	3.74e-15
5.76e+13	3.09e-15	4.77e-15	8.45e-15	1.93e-14	1.95e-14	4.93e-15
1.15e+14	2.89e-15	7.04e-15	4.94e-15	8.43e-15	8.46e-15	7.21e-15
2.30e+14	3.07e-15	5.87e-15	3.62e-15	5.91e-15	6.91e-15	6.35e-15
4.61e+14	4.16e-15	6.32e-15	4.32e-15	4.69e-15	5.70e-15	6.49e-15

IV. CONCLUSIONS

Hydrogen-cesium combined timekeeping is a common method, which aims at taking into account the excellent characteristics of two kinds of atomic clocks. In this paper, we first propose a fusion method based on the scale level of the hydrogen masers and the cesium clock ensemble. First, the exponential filtering method is used to generate the time scale of hydrogen masers, and the weighted average algorithm is used to generate the time scale of cesium clock ensemble. Secondly, the V-C filtering method is used to calculate the fusion time scale. In this paper, the measured data of atomic clock is used to test the effectiveness of the algorithm. The calculated results of measured data show that V-C combined smoothing method not only reduces the noise but also ensures the fidelity of the time scale of cesium clocks. It effectively utilizes the excellent short-term stability of hydrogen masers and the excellent long-term stability of cesium clocks. The stability of fusion time scale produced by V-C combined smoothing method is better than that produced by two types of atomic clocks and the weighted average method AT1 and ALGOS respectively. However, all filters have the problem of boundary effect, which will be further studied.

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